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HEAT CONDUCTION AND HEAT EXCHANGE IN TECHNOLOGICAL PROCESSES

COMPUTATIONAL ANALYSIS OF THE REGIMES OF SOLIDIFICATION AND COOLING OF A CONTINUOUS CASTING WITH A CIRCULAR CROSS SECTION

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The nonlinear problem on heat conduction with discontinuity coefficients was solved by the finite-difference method with the use of a grid irregular in time and space. The calculations were carried out using difference schemes with a "weight," making it possible to control the calculation accuracy. The computational experiment has been performed for a steel of standard quality.

Introduction. At present, the main challenge of producers of metals is the obtaining of high-quality castings having a low cost. This can be attained first of all by increasing the efficiency of equipment used for production of a metal, which is determined by the yield of the finished metal, the rate of casting, the quality of the production process, and the design of the units and mechanisms of the equipment. The problem of improvement of the indicated characteristics can be solved with the use of the modern mathematical apparatus that allows one to determine the parameters of the process of production of a metal and the design parameters of a continuous-casting machine (CCM) used in this process, which mainly determine the quality of the finished metal and the efficiency of the CCM [1–5].

The main problem of the mathematical simulation of the process of production of a casting is the construction of its temperature field on the basis of the input characteristics at different instants of crystallization and cooling and at the instant of unbending of the ingot at a definite temperature of its surface. The solution of such problems by empirical methods is expensive and time-consuming. Moreover, corresponding experiments should be multi-variant because many factors influence the production of a steel casting — the rate of extending of an ingot, the cross-section dimension of the ingot and its initial temperature, the chemical composition of the steel, and the geometric characteristics of a CCM used in the production process.

The aim of the present work is to solve the nonlinear problem on heat conduction in a solidifying casting with the use of an efficient numerical method and to perform a computational experiment for a steel of characteristic quality and for a standard industrial CCM. The finite-difference method was used for approximate solution of this problem. The difference scheme used in the calculations was constructed by the integro-interpolation method [6]. Calculations were carried out on a grid irregular in time and space. Unlike, e.g., [5], we used models and numerical algorithms accounting for the discontinuities of the heat conduction and density of the steel arising as a result of phase transitions as well as difference schemes with "weights," making it possible to control the accuracy of calculations.

Physicomathematical Model. The heat conduction in a steel ingot is defined by the nonstationary, nonlinear heat-conduction equation

$$\rho (T)\, c_{\text{ef}} (T) \frac{\partial T}{\partial t} = \text{div}\, (\lambda (T)\, \text{grad}\, (T)) . \tag{1}$$

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It is assumed that the steel is crystallized at temperatures falling within a fairly wide range $\Delta T_{cr} = T_{sol} - T_{liq}$. Consequently, impurity-free metals crystallized at a temperature close to T_{cr} will not be considered. It is also assumed that the local overcooling of the melt can be disregarded.

The accuracy of solution of the problem on the heat conduction in a steel depends significantly on the temperature dependence of the nonlinearity of the thermophysical characteristics of the steel. Because of this, we will formulate the problem using known methods of calculating the heat-conductivity coefficient and the density of materials [7–10]. It should be noted that the functions of these thermophysical characteristics have discontinuities of the first kind.

The shape of the cross section of the castings being investigated and their axial symmetry allow us to pass from the Cartesian coordinate system to the cylindrical coordinate system and use the following equation:

$$\rho(T)\, c_{ef}(T) \frac{\partial T}{\partial t} = \frac{1}{r} \frac{\partial}{\partial r}\left(r\lambda(T) \frac{\partial T}{\partial r} \right), \quad 0 < r < R, \quad 0 < t < t_f . \tag{2}$$

It is assumed that the heat exchange at the inner wall (in the cooling zones) of the mould of an ingot proceeds by the Newton convection law

$$-\lambda(T) \frac{\partial T}{\partial r}\bigg|_{r=R} = \alpha \left(T\big|_{r=R} - T_{med}\right), \tag{3}$$

and a heat flow has discontinuities of the first kind at the instant a melt passes from one zone to an other one. Because of this, the solution of the problem at the interface between the zones is not smooth in character — the first time derivatives of the solutions at this interface have discontinuities of the first kind.

It is assumed that the temperature field at the symmetry axis ($r = 0$) of a casting is bounded [11]:

$$\lim_{r \to 0} r\lambda(T) \frac{\partial T}{\partial r} = 0 . \tag{4}$$

Unlike, e.g., [12] where the condition $\left.\frac{\partial T}{\partial r}\right|_{r=0} = 0$ was used, relation (4) defines the behavior of the solution in the vicinity of the axis of the casting more exactly. This boundary condition allows one, on the one hand, to obtain a more exact solution of the problem and, on the other, to construct a difference boundary condition for Eq. (2) at nodes in the neighborhood of the point $r = 0$ and retain the thermal balance of the numerical method (the conservatism of the difference schemes) throughout the range of space integration. Conditions (2)–(4) are supplemented with the initial condition

$$T\big|_{t=0} = T_m . \tag{5}$$

Numerical Algorithm. Problem (2)–(5) is solved approximately by the finite-difference method with the use of a grid irregular in time and space.

For the region $\Omega = [(0;\, R) \times (0;\, t_f]$ we will introduce an irregular spatial grid ω_r:

$$\omega_r = \left\{ r_i \cup [0;\, R], \quad r_i = r_0 + \sum_{k=1}^{i} h_k, \quad i = 1, 2, ..., N, \quad r_0 = 0, \quad r_N = R \right\}$$

and an irregular time grid ω_t:

$$\omega_t = \left\{ t_j \cup [0;\, t_f], \quad t_j = t_0 + \sum_{k=1}^{j} \tau_k, \quad j = 1, 2, ..., M, \quad t_0 = 0, \quad t_N = t_f \right\},$$

where r_i is a node of the grid ω_r; t_j is a node of the grid ω_t; h_i is a pitch of the grid ω_r, $i = 1, 2, ..., N$; τ_j is a pitch of the grid ω_t, $j = 1, 2, ..., M$; $\omega = \omega_r \times \omega_t$.

We introduce the following designations. Let $y = y_i^j = y(r_i, t_j)$ be a function determined in the grid ω., and

$$y_{\hat{x}} = \left(y_{i+1}^j - y_i^j\right) / \hbar_i,\ \hbar_i = 0.5(h_i + h_{i+1}),\ i = 1, 2, ..., N-1, j = 0, 1, ..., M\ ;\ y_{\bar{x}} = \left(y_i^j - y_{i-1}^j\right) / h_i,\ i = 1, 2, ..., N,$$

$$j = 0, 1, ..., M\ ;\ \check{y} = y_i^{j-1} = y\,(x_i, t_{j-1})\ ,\quad y_{\bar{t}} = \left(y_i^j - y_i^{j-1}\right) / \tau_j\ ,\quad i = 0, 1, ..., N\ ,\quad j = 1, 2, ..., M\ .$$

A difference scheme for Eq. (2) is constructed with a weight σ in accordance with [6]:

$$\left(\sigma\tilde{c}\,(y_i) + (1-\sigma)\,\tilde{c}\,(\check{y}_i)\right) y_{\bar{t}} = \frac{1}{r_i}\left(\sigma\Lambda\,(y_i) + (1-\sigma)\,\Lambda\,(\check{y}_i)\right),\ 0 \le \sigma \le 1\ ,\ i = 1, 2, ..., N-1, j = 1, 2, ..., M\ , \tag{6}$$

where

$$\tilde{c}\,(y_i) = \rho\,(y_i^j)\,c_{\rm ef}\,(y_i^j)\ ;\quad \Lambda\,(y_i) = \frac{1}{\hbar_i}\left(a_{i+1}^j\,\frac{y_{i+1}^j - y_i^j}{h_{i+1}} - a_i^j\,\frac{y_i^j - y_{i-1}^j}{h_i}\right),\quad a_i^j = \frac{1}{2}\left(r_i\,\lambda\,(y_i^j) + r_{i-1}\,\lambda\,(y_{i-1}^j)\right).$$

The boundary conditions (3) and (4) are approximated with the same order as Eq. (2):

$$-\sigma a_N^j\,\frac{y_N^j - y_{N-1}^j}{h_N} - (1-\sigma)\,a_N^{j-1}\,\frac{y_N^{j-1} - y_{N-1}^{j-1}}{h_N} =$$

$$= \frac{h_N}{2}\,R\left(\sigma\tilde{c}\,(y_N) + (1-\sigma)\,\tilde{c}\,(\check{y}_N)\right)\frac{y_N^j - y_N^{j-1}}{\tau_j} + \alpha R\left(\sigma y_N^j + (1-\sigma)\,y_N^{j-1}\right) - \alpha R T_{\rm med}\ , \tag{7}$$

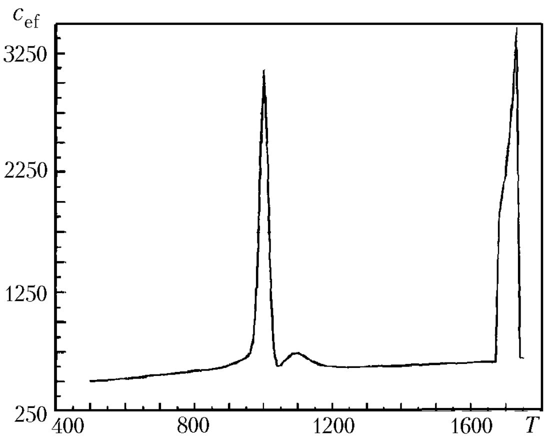
$$\sigma\tilde{a}_0^j\,\frac{y_1^j - y_0^j}{h_0} + (1-\sigma)\,\tilde{a}_0^{j-1}\,\frac{y_1^{j-1} - y_0^{j-1}}{h_1} = \frac{h_1}{4}\left(\sigma\tilde{c}\,(y_0) + (1-\sigma)\,\tilde{c}\,(\check{y}_0)\right)\frac{y_0^j - y_0^{j-1}}{\tau_j},\ \tilde{\alpha}_0^j = \frac{1}{2}\left(\lambda\,(y_0^j) + \lambda\,(y_1^j)\right). \tag{8}$$

The weight σ can take values from the interval [0; 1]. In practice, the following values of it are used: 0 or 1, or 0.5. A Crank–Nicholson scheme with an approximation error $O(h^2 + \tau^2)$ is obtained at σ = 0.5, an asymptotically stable scheme with an approximation error $O(h^2 + \tau)$ is obtained at σ = 1, and an explicit difference scheme with an error $O(h^2 + \tau)$ is obtained at σ = 0.

The system of nonlinear algebraic equations (6)–(8) is solved in the obvious way in the case where σ = 0 and by the iteration method in the case where $\sigma \ne 0$ [6]. Calculations with discontinuous heat-conductivity and density coefficients are carried out by the methods described in [13, 14]. To obtain reliable results, it is necessary to perform calculations for different values of σ and vary the pitch of the grid ω in their process.

Computational Experiment. The problem on heat conduction in a solidifying steel ingot of quality 70 K obtained by the continuous-casting method on a standard CCM was numerically solved. The initial data for solution of the problem were as follows: the radius of the casting $R = 0.155$ m, the rate of casting is 0.75 m/min, the initial temperature of the melt $T_{\rm m}$ = 1768 K, the temperature of the liquidus $T_{\rm liq}$ = 1738 K, the temperature of the solidus $T_{\rm sol}$ = 1679 K, the temperature of the medium $T_{\rm med}$ = 303 K, the specific heat

$$c_{\rm ef}\,(T) = \begin{cases} c_{\rm liq} & \text{at}\quad T \ge T_{\rm liq}\ , \\ c_{\rm sol} - L\,\dfrac{d\Psi}{dT} & \text{at}\quad T_{\rm sol} < T < T_{\rm liq}\ , \\ c_{\rm sol} & \text{at}\quad T \le T_{\rm sol}\ , \end{cases}$$



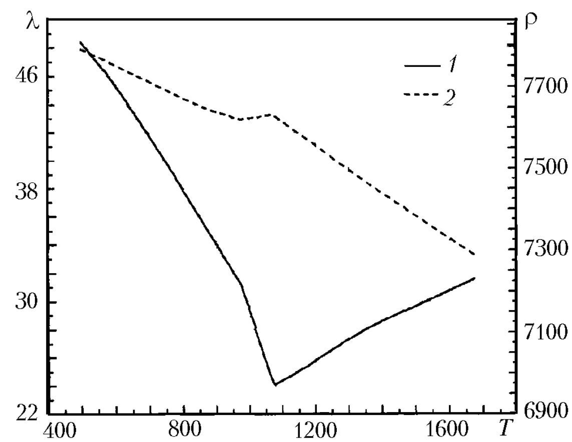


Fig. 1. Temperature dependence of the specific heat of a casting. c_{ef}, J/(kg·K); T, K.

Fig. 2. Temperature dependences of the heat-conductivity coefficient (1) and the density (2) of the casting. T, K; λ, W/(m·K); ρ, kg/m^3.

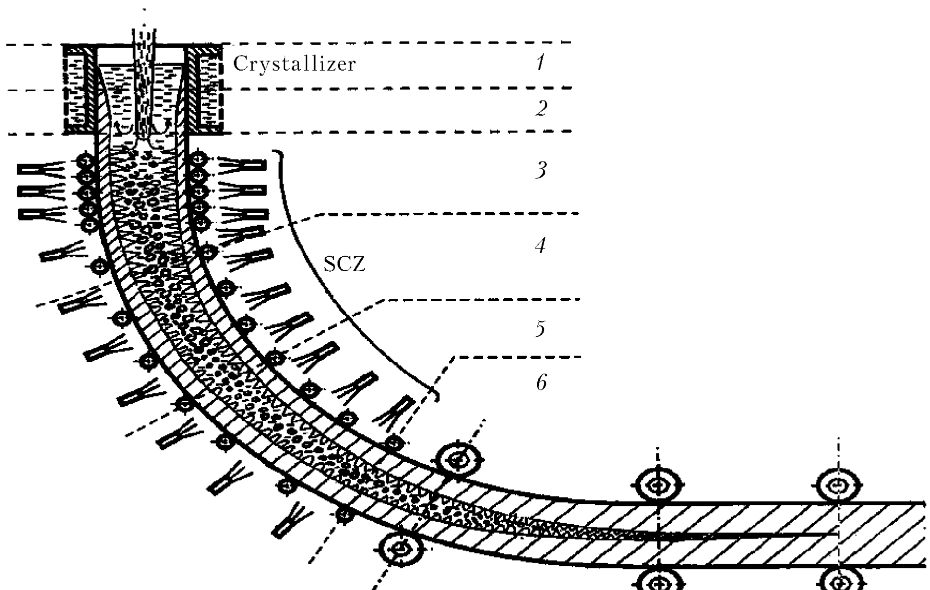


Fig. 3. Diagram of a CCM-3.

where L = 272 kJ/kg; c_{liq} = 710 J/(kg·K); $c_{sol} = 496 + 0.16(T - 323) + \sum_{i=1}^{4} \frac{m_i L_i}{\delta T_i} \exp\left[-a_1^2\left(\frac{T_i - T}{\delta T_i}\right)^2\right]$, where $a_1 = 4$, $m_i = 4.5141$, $i = \overline{1, 4}$; T_1 = 1000 K, T_2 = 1033 K, T_3 = 923 K, T_4 = 1033 K; δT_1 = 70 K, δT_2 = 350 K, δT_3 = 1100 K, δT_4 = 170 K; $L_1 = 44{,}076 - 85{,}622x^2 + 50{,}357x$, $L_2 = 5163.2 - 74{,}009x^2 + 70{,}232x$, $L_3 = 2622.3 - 92{,}590x^2 + 80{,}523x$, $L_4 = 14{,}775 + 154{,}544x^2 - 142{,}489x$; $\Psi(T) = c^*_{liq} - x/(c^*_{liq} - c^*_{sol})$, $x = 0.7$; $c^*_{liq}(T) = 15.463359 - (0.124528\times 10^{-1})T + (0.216279\cdot 10^{-5})T^2$, $c^*_{sol} = -11.0388 + (0.278656\cdot 10^{-1})T - (0.120163\cdot 10^{-4})T^2$.

Figure 1 shows the temperature dependence of the specific heat of the steel being investigated. The dependences of the density of the steel and its heat-conductivity coefficient on the temperature are shown in Fig. 2. The CCM is conditionally divided, in accordance with its design, into five zones (Fig. 3) differing in the intensity of the external cooling. The crystallizer includes two zones: the zone of contact of the melt with the wall of the crystallizer

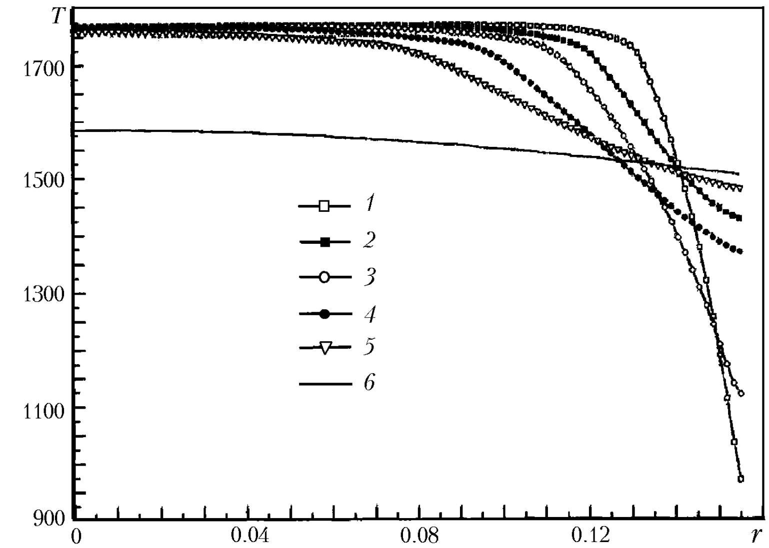


Fig. 4. Temperature distribution over the cross section of the ingot at the output of the zones of the CCM (1–6 are numbers of zones): l = 0.4 and α = 2100 (1), 0.4 and 60 (2), 0.47 and 850 (3), 0.95 and 120 (4), 1.51 and 40 (5), 18.97 m and 25 W/(m^2·K) (6). T, K; r, m.

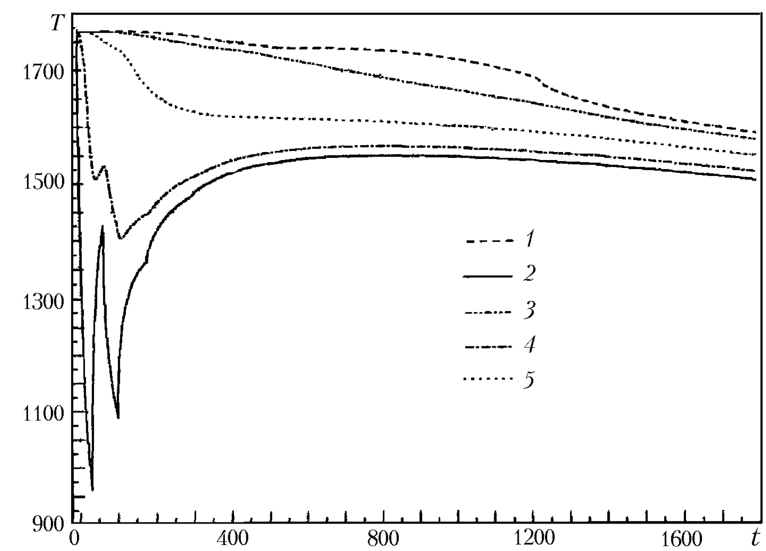


Fig. 5. Change in the temperature of the casting in the process of its solidification: 1) axis of the ingot (r = 1); 2) surface of the ingot (r = 0.155); 3) distance of 100 mm from the surface of the ingot (r = 0.055); 4) distance of 15 mm from the surface of the ingot (r = 0.140); 5) distance of 50 mm from the surface of the ingot (r = 0.105 m). T, K; t, sec.

1 (l = 0.4 m) and the zone of the gas-air gap 2 (l = 0.4 m). After the continuous casting leaves the crystallizer, it passes through four zones: the zone of intensive injector cooling 3 (l = 0.47 m), the secondary-cooling zones (SCZ) 4 and 5 (l = 0.95 and 1.51 m respectively), and the zone of cooling in air 6 (l = 18.97 m).

The CCM zones are characterized not only by the length but also by the heat-transfer coefficient α. In the calculations, a fairly large coefficient of external heat exchange (α = 2100 W/(m^2·K)) was used for zone 1 because, in this zone, a contact heat exchange between the surface of the casting and the inner wall of the crystallizer is realized. For zone 2, in which there occurs a radiative heat exchange between the surface of the casting and the inner surface of the crystallizer and heat is transferred through the thin gap filled with a gas mixture and the scale formed on the surface of the casting, the average heat-transfer coefficient is taken to be equal to 60 W/(m^2·K). In zone 3, where the strongest injector cooling is realized, the heat-transfer coefficient is equal to 850 W/(m^2·K). In zone 4, the cooling water can fall on the surface of the casting; therefore, the heat-transfer coefficient is taken to be equal to 120 W/(m^2·K) in this zone. In zone 5, the temperature of the surface of the casting and the intensity of cooling are somewhat lower than in the previous regions of the secondary-cooling zone; therefore, the heat-transfer coefficient is taken to be equal to 40 W/(m^2·K) in this zone. In the case of cooling in air (zone 6), the heat lost through the surface of the casting is relatively small and accounts for 25 W/(m^2·K).

As a result of the numerical solution of the problem, Eqs. (2)–(5), it was established that the steel ingot being investigated is completely crystallized in 20.5 min. Below are graphs characterizing the temperature distribution over the section of the ingot at the output of the CCM zones (Fig. 4) and the graphs of change in the temperature of the ingot in the process of its cooling, obtained for different values of r (Fig. 5).

In [5], experimental temperature distributions, measured for a rectangular (250 × 300 mm) steel ingot of quality 70 K, obtained on the CCM-3 of the Republican Unitary Enterprise "Belarusian Engine Works," are presented. The deviation of the calculated time of solidification of the ingot from the experimental time is 6.5%. In our opinion, this difference is due to the

a) measurement errors of detectors;

b) approximate values of the coefficients of the equation used in the calculations;

c) difference in shape between the castings investigated;

d) possible difference in chemical composition between the steel ingots investigated;

e) tolerances in the specifications of the process;

f) approximation errors involved in the numerical method used for calculations and the errors of the calculations themselves.

It should be noted that, despite the difference in shape between the castings investigated, the temperature distributions presented in Fig. 4 are close to the experimental temperature distributions.

Conclusions. The method proposed for calculating the temperature fields of a solidifying steel ingot will be useful for a multi-variant simulation in the case where it is necessary to obtain results fairly rapidly without regard for the fact that the real process is multidimensional.

NOTATION

$c_{ef}(T)$, specific heat, J/(kg·K); c_{liq} and c_{sol}, specific heat of the liquidus and solidus, J/(kg·K); L, specific heat of crystallization, J/kg; l, length of a zone, m; r, distance from the center of a casting, m; R, radius of the casting, m; t and t_f, current and finite instants of time, sec; T, temperature, K; T_{cr}, crystallization temperature, K; T_m, initial temperature of the melt, K; T_{liq}, T_{sol}, and T_{med}, temperature of the liquidus, solidus, and medium, K; $y(r_i, t_j)$, function determined on the grid; v, rate of casting of a metal, m/min; α, coefficient of external heat exchange, W/(m^2·K); $\lambda(T)$, heat-conductivity coefficient, W/(m·K); $\rho(T)$, density, kg/m^3; ω_r, spatial grid; ω_t, time grid; Ψ, relative fraction of the liquid phase in the control volume of the solidifying melt. Subscripts: ef, effective; liq, liquidus; sol, solidus; f, finite; m, melt; med, medium; cr, crystallization.

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